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TRACE ANOMALY INDUCED EFFECTIVE ACTION AND HAWKING RADIATION FOR 2d DILATONIC SUPERGRAVITY

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ABSTRACT

We construct the theory of 2d dilatonic supergravity(SG) with matter and dilaton supermultiplets coupled to dilaton functions. Trace anomaly and induced effective action for matter supermultiplet are calculated (what gives also large-N effective action for dilatonic SG). Study of black holes and Hawking radiation which turns out to be zero in supersymmetric CGHS model with dilaton coupled matter is presented. In the same way one can study spherically symmetric collapse for other 4d SG using simplified 2d approach.

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There are different motivations to study 2d black holes [1]. First of all, such models are string inspired ones. Second, using spherically symmetric reduction anzats of 4d Einstein-scalar theory one is left with the action of 2d dilatonic gravity with dilaton coupled matter [2]. Hence, 4d spherical collapse maybe understood in terms of 2d dilatonic gravity. There were recently attempts to study trace anomaly and induced effective action for 2d dilaton coupled scalars [3, 4, 5] (what was motivated by appearence of dilaton coupled scalars after reduction) as well as related research of 2d black holes and Hawking radiation with account of quantum effects of dilaton coupled scalars.

It would be of interest to present supersymmetric generalization of such investigation as spherically symmetric reduction of 4d SG with matter leads to 2d dilatonic SG with dilaton coupled matter supermultiplet. That will be the purpose of present work- to find anomaly induced effective action in 2d dilatonic SG with matter.

At first we are going to construct the action of 2d dilatonic supergravity with dilaton supermultiplet and with matter supermultiplet. In order to construct the Lagrangian of two-dimensional dilatonic supergravity, we use the component formulation of ref.[6] (for introduction, see book[7]).

In this paper, all the scalar fields are real and all the spinor fields are Majorana spinors.

We introduce dilaton multiplet $\Phi = (\phi, \chi, F)$ and matter multiplet $\Sigma_i = (a_i, \chi_i, G_i)$, which has the conformal weight $\lambda = 0$, and the curvature multiplet W

$$W = \left(S, \eta, -S^2 - \frac{1}{2}R - \frac{1}{2}\bar{\psi}^\mu \gamma^\nu \psi_{\mu\nu} + \frac{1}{4}\bar{\psi}^\mu \psi_\mu\right) \ . \tag{1}$$

Here R is the scalar curvature, for other notations see [6].

Then the general action of 2d dilatonic supergravity is given in terms of general functions of the dilaton $C(\phi)$, $Z(\phi)$, $f(\phi)$ and $V(\phi)$ as follows

$$\begin{aligned}
\mathcal{L} &= -\left[C(\Phi) \otimes W\right]_{\rm inv} + \left[V(\Phi)\right]_{\rm inv} \\
&+ \frac{1}{2}\left[\Phi \otimes \Phi \otimes T_P(Z(\Phi))\right]_{\rm inv} - \left[Z(\Phi) \otimes \Phi \otimes T_P(\Phi)\right]_{\rm inv} \\
&+ \sum_{i=1}^{N} \left\{ \frac{1}{2}\left[\Sigma_i \otimes \Sigma_i \otimes T_P(f(\Phi))\right]_{\rm inv} - \left[f(\Phi) \otimes \Sigma_i \otimes T_P(\Sigma_i)\right]_{\rm inv} \right\} \ . \qquad (2)
\end{aligned}$$

$T_P(Z)$ is called the kinetic multiplet and $[Z]_{\rm inv}$ expresses the invariant Lagrangian. Hence, we constructed the classical action for 2d dilatonic supergravity with dilaton and matter supermultiplets.

We now study the trace anomaly and effective action in large-N approximation for the 2d dilatonic supergravity. We consider only bosonic background below as it will be sufficient for our purposes (study of black hole type solutions).

On the bosonic background where the dilatino χ and the Rarita-Schwinger fields vanish, one can show that the gravity and dilaton part of the Lagrangian have the following form:

$$\begin{aligned}
&[C(\Phi)\otimes W]_{\rm inv} = e\left[-C(\phi)\left(S^2+\frac{1}{2}R\right)-C'(\phi)FS\right] \ , \\
&[\Phi\otimes\Phi\otimes T_P(Z(\Phi))]_{\rm inv} = e\left[\phi^2\Box(Z(\phi))+2Z'(\phi)\phi F^2\right] \ , \\
&[Z(\Phi)\otimes\Phi\otimes T_P(\Phi)]_{\rm inv} = e\left[Z(\phi)\phi\Box\phi+Z'(\phi)\phi F^2+Z(\phi)F^2\right] \ , \\
&[V(\Phi)]_{\rm inv} = e\left[V'(\phi)F+SV(\phi)\right] \ .
\end{aligned} \tag{3}$$

For the matter part we obtain

$$\begin{aligned}
&[f(\Phi)\otimes\Sigma_i\otimes T_P(\Sigma_i)]_{\rm inv} = e\left[f(\phi)(a_i\Box a_i-\bar\xi_i\tilde{\not{D}}\xi_i)+f'(\phi)Fa_iG_i+f(\phi)G_i^2\right] \\
&[\Sigma_i\otimes\Sigma_i\otimes T_P(f(\Phi))]_{\rm inv} = e\left[a_i^2\Box(f(\phi))+2f'(\phi)Fa_iG_i\right] \ .
\end{aligned} \tag{4}$$

Using the equations of motion with respect to the auxilliary fields S, F, G_i, on the bosonic background one can show that

$$S=\frac{C'(\phi)V'(\phi)-2V(\phi)Z(\phi)}{C'^2(\phi)+4C(\phi)Z(\phi)}, \ F=\frac{C'(\phi)V(\phi)+2C(\phi)V'(\phi)}{C'^2(\phi)+4C(\phi)Z(\phi)}, \ G_i=0. \tag{5}$$

We will be interested later in the supersymmetric extension [10] of the CGHS model [9] as in specifical example for study of black holes and Hawking radiation. For such a model

$$C(\phi)=2{\rm e}^{-2\phi} \ , \quad Z(\phi)=4{\rm e}^{-2\phi} \ , \quad V(\phi)=4{\rm e}^{-2\phi} \ , \tag{6}$$

we find

$$S=0 \ , \quad F=-\lambda \ , \quad G_i=0 \ . \tag{7}$$

Using (4), we can show that on bosonic background

$$\sum_{i=1}^{N}\left\{\frac{1}{2}\left[\Sigma_i\otimes\Sigma_i\otimes T_P(f(\Phi))\right]_{\rm inv}-\left[f(\Phi)\otimes\Sigma_i\otimes T_P(\Sigma_i)\right]_{\rm inv}\right\} \tag{8}$$
$$=ef(\phi)\sum_{i=1}^{N}(g^{\mu\nu}\partial_\mu a_i\partial_\nu a_i+\bar{\xi}_i\gamma^\mu\partial_\mu\xi_i-f(\phi)G_i^2)+\left(\begin{array}{c}\text{total divergence}\\ \text{terms}\end{array}\right)$$

Here we have used the fact that $\bar{\xi}_i\gamma_5\xi=0$ for the Majorana spinor ξ_i.

Let us start now the investigation of effective action in above theory. It is clearly seen that theory (8) is conformally invaiant on the gravitational background under discussion. Then using standard methods, we can prove that theory with matter multiplet Σ_i is superconformally invariant theory. First of all, one can find the trace anomaly T for the theory (8) on gravitational background using the following relation

$$\Gamma_{div}=\frac{1}{n-2}\int d^2x\sqrt{g}b_2\ ,\qquad T=b_2 \tag{9}$$

where b_2 is b_2 coefficient of Schwinger-De Witt expansion and Γ_{div} is one-loop effective action. The calculation of Γ_{div} (9) for the quantum theory with Lagrangian (8) has been done some time ago in ref.[3]. Using results of this work, we find

$$T\ =\ \frac{1}{24\pi}\left\{\frac{3}{2}NR-3N\left(\frac{f''}{f}-\frac{f'^2}{2f^2}\right)(\nabla^\lambda\phi)(\nabla_\lambda\phi)-3N\frac{f'}{f}\Delta\phi\right\}\ . \tag{10}$$

It is remarkable that Majorana spinors do not give the contribution to the dilaton dependent terms in trace anomaly as it was shown in [3]. They only alter the coefficient of curvature term in T (10). Hence, except the coefficient of curvature term in T (10), the trace anomaly (10) coincides with the correspondent expression for dilaton coupled scalar [4]. Note also that for particular case $f(\phi)=\mathrm{e}^{-2\phi}$ the trace anomaly for dilaton coupled scalar has been recently calculated in refs.[5].

Making now the conformal transformation of the metric $g_{\mu\nu}\rightarrow\mathrm{e}^{2\sigma}g_{\mu\nu}$ in the trace anomaly, and using relation:

$$T=\frac{1}{\sqrt{g}}\frac{\delta}{\delta\sigma}W[\sigma] \tag{11}$$

one can find anomaly induced action $W[\sigma]$. In the covariant, non-local form it may be found as following:

$$W = -\frac{1}{2}\int d^2x\sqrt{g}\left[\frac{N}{32\pi}R\frac{1}{\Delta}R - \frac{N}{16\pi}\frac{f'^2}{f^2}\nabla^\lambda\phi\nabla_\lambda\phi\frac{1}{\Delta}R - \frac{N}{8\pi}\ln fR\right] \ . \qquad (12)$$

Hence, we got the anomaly induced effective action for dilaton coupled matter multiplet in the external dilaton-gravitational background. We should note that the same action W (12) gives the one-loop large-N effective action in the quantum theory of supergravity with matter (2) (i.e., when all fields are quantized).

We can now rewrite W in a supersymmetric way. In order to write down the effective action expressing the trace anomaly, we need the supersymmetric extention of $\frac{1}{\Delta}R$. The extension is given by using the inverse kinetic multiplet in [8], or equivalently by introducing two auxiliary field $\Theta = (t, \theta, T)$ and $\Upsilon = (u, \upsilon, U)$. We can now construct the following action

$$[\Theta\otimes(T_P(\Upsilon) - W)]_{\rm inv} \ . \qquad (13)$$

The Θ-equation of motion tells that, in the superconformal gauge

$$e^a_\mu = {\rm e}^\rho\delta^a_\mu \ (e = {\rm e}^\rho) \ , \quad \psi_\mu = \gamma_\mu\psi \ (\bar\psi_\mu = -\bar\psi\gamma_\mu) \qquad (14)$$

we find

$$u \sim \rho \sim -\frac{1}{2\Delta}R \ , \quad \upsilon \sim \psi \ . \qquad (15)$$

Then we obtain

$$\begin{aligned}
&\sqrt{g}R\frac{1}{\Delta}R \sim 4\,[W\otimes\Upsilon]_{\rm inv} \\
&\sim -\left[\Sigma_i\otimes\Sigma_i\otimes T_P\left(\left(\frac{f'^2(\Phi)}{f(\Phi)} - f''(\Phi)\right)\right)\Upsilon\right]_{\rm inv} \\
&\qquad +2\left[\left(\frac{f'^2(\Phi)}{f(\Phi)} - f''(\Phi)\right)\otimes\Upsilon\otimes\Sigma_i\otimes T_P(\Sigma_i)\right]_{\rm inv} \\
&\sqrt{g}\frac{f'^2(\phi)}{f^2(\phi)}\nabla_\lambda\phi\nabla^\lambda\phi\frac{1}{\Delta}R \\
&\sim -\left[\Phi\otimes\Phi\otimes T_P\left(\frac{f'^2(\Phi)}{f^2(\Phi)}\otimes\Upsilon\right)\right]_{\rm inv} + 2\left[\frac{f'^2(\Phi)}{f^2(\Phi)}\otimes\Upsilon\otimes\Phi\otimes T_P(\Phi)\right]_{\rm inv} \\
&\sqrt{g}\ln f(\phi)\,R \sim -2\,[\ln f(\Phi)\otimes W]_{\rm inv} \ . \qquad (16)
\end{aligned}$$

That finishes the construction of large-N effective action for 2d dilatonic supergravity with matter in supersymmetric form.

We now discuss the particular 2d dilatonic supergravity model which represents the supersymmetric extension of CGHS model. Note that as a matter we use dilaton coupled matter supermultiplet. We would like to estimate back-reaction of such matter supermultiplet to black holes and Hawking radiation working in large-N approximation. Since we are interesting in the vacuum (black hole) solution, we consider the background where matter fields, the Rarita-Schwinger field and dilatino vanish.

In the superconformal gauge the equations of motion can be obtained by the variation over $g^{\pm\pm}$, $g^{\pm\mp}$ and ϕ

$$
\begin{aligned}
& 0 = T^c_{\mu\nu} + T^q_{\mu\nu}, \qquad 0 = \mathcal{P}^c + \mathcal{P}^q \\
& T^c_{\pm\pm} = \mathrm{e}^{-2\phi}\left(4\partial_\pm\rho\partial_\pm\phi - 2\left(\partial_\pm\phi\right)^2\right) \\
& T^q_{\pm\pm} = \frac{N}{8}\left(\partial_\pm^2\rho - \partial_\pm\rho\partial_\pm\rho\right) \\
& +\frac{N}{8}\left\{\left(\partial_\pm h(\phi)\partial_\pm h(\phi)\right)\rho + \frac{1}{2}\frac{\partial_\pm}{\partial_\mp}\left(\partial_\pm h(\phi)\partial_\mp h(\phi)\right)\right\} \\
& +\frac{N}{8}\left\{-2\partial_\pm\rho\partial_\pm h(\phi) + \partial_\pm^2 h(\phi)\right\} + \frac{N}{64}\frac{\partial_\pm}{\partial_\mp}\left(h'(\phi)^2F^2\right) + t^\pm(x^\pm) \\
& T^c_{\pm\mp} = \mathrm{e}^{-2\phi}\left(2\partial_+\partial_-\phi - 4\partial_+\phi\partial_-\phi - \lambda^2\mathrm{e}^{2\rho}\right) \\
& T^q_{\pm\mp} = -\frac{N}{8}\partial_+\partial_-\rho - \frac{N}{16}\partial_+h(\phi)\partial_-\tilde{\phi} - \frac{N}{8}\partial_+\partial_-h(\phi) \\
& -\frac{N}{64}h'(\phi)F^2 + \left(\frac{N}{16}US + \frac{N}{2}(-h(\phi)S^2h'(\phi)FS\right)\mathrm{e}^{2\rho} \\
& \mathcal{P}^c = \mathrm{e}^{-2\phi}\left(-4\partial_+\partial_-\phi + 4\partial_+\phi\partial_-\phi + 2\partial_+\partial_-\rho + \lambda^2\mathrm{e}^{2\rho}\right) \\
& \mathcal{P}^q = -\frac{Nf'}{f}\left\{\frac{1}{16}\partial_+(\rho\partial_-h(\phi)) + \frac{1}{16}\partial_-(\rho\partial_+h(\phi)) - \frac{1}{8}\partial_+\partial_-\rho\right\}\ . \qquad (17)
\end{aligned}
$$

Here $h(\phi) \equiv \ln f(\phi)$ and $t^\pm(x^\pm)$ is a function which is determined by the boundary condition. Note that there is, in general, a contribution from the auxilliary fields to $T_{\pm\mp}$ besides the contribution from the trace anomaly.

In large-N limit, where classical part can be ignored, the field equations become simpler

$$
0 = T^q_{\mu\nu}\ , \qquad 0 = \mathcal{P}^q\ . \qquad (18)
$$

Here we used the Θ-equation and the equations for the auxilliary fields S and F. The function $t^{\pm}(x^{\pm})$ in (18) can be absorbed into the choice of the coordinate and we can choose $t^{\pm}(x^{\pm}) = 0$. We can show that the general solutions of (18) are given by

$$h(\phi) = \int d\rho \frac{1 \pm \sqrt{1+\rho}}{\rho} \; , \qquad \rho = -1 + \left(\rho^{+}(x^{+}) + \rho^{-}(x^{-})\right)^{\frac{2}{3}} \; . \tag{19}$$

Here $\rho^{\pm}$ is an arbitrary function of $x^{\pm} = t \pm x$. The scalar curvature is given by

$$R = 8\mathrm{e}^{-2\rho}\partial_{+}\partial_{-}\rho = -\frac{4\mathrm{e}^{-2\left\{-1+\left(\rho^{+}(x^{+})+\rho^{-}(x^{-})\right)^{\frac{2}{3}}\right\}}\rho^{+\prime}(x^{+})\rho^{-\prime}(x^{-})}{(\rho^{+}(x^{+}) + \rho^{-}(x^{-}))^{\frac{4}{3}}} \; . \tag{20}$$

Note that when $\rho^{+}(x^{+}) + \rho^{-}(x^{-}) = 0$, there is a curvature singularity. Especially if we choose

$$\rho^{+}(x^{+}) = \frac{r_0}{x^{+}} \; , \quad \rho^{-}(x^{-}) = -\frac{x^{-}}{r_0} \tag{21}$$

there are curvature singularities at $x^{+}x^{-} = r_0^2$ and horizon at $x^{+} = 0$ or $x^{-} = 0$. Hence we got black hole solution in the model under discussion. The asymptotic flat regions are given by $x^{+} \to +\infty$ $(x^{-} < 0)$ or $x^{-} \to -\infty$ $(x^{+} > 0)$.

We now consider the Hawking radiation in the bosonic background where the fermionic fields vanish. We investigate the case that

$$f(\phi) = \mathrm{e}^{\alpha\phi} \quad (h(\phi) = \alpha\phi) \; . \tag{22}$$

Substituting the classical black hole solution which appeared in the original CGHS model [9]

$$\rho = -\frac{1}{2}\ln\left(1 + \frac{M}{\lambda}\mathrm{e}^{\lambda(\sigma^{-}-\sigma^{+})}\right) \; , \qquad \phi = -\frac{1}{2}\ln\left(\frac{M}{\lambda} + \mathrm{e}^{\lambda(\sigma^{+}-\sigma^{-})}\right) \tag{23}$$

(Here M is the mass of the black hole and we used asymptotic flat coordinates.) into the quantum part of the energy momentum tensor $T^{q}_{\mu\nu}$ in (18) and using eq.(7), we find the explicit expressions for quantum energy-momentum tensor.

Then when $\sigma^+ \to +\infty$, the energy momentum tensor behaves as

$$T^q_{+-} \to 0 \ , \quad T^q_{\pm\pm} \to \frac{N\lambda^2}{16}\alpha^2 + t^\pm(\sigma^\pm) \ . \tag{24}$$

In order to evaluate $t^\pm(\sigma^\pm)$, we impose a boundary condition that there is no incoming energy. This condition requires that T^q_{++} should vanish at the past null infinity ($\sigma^- \to +\infty$) and if we assume $t^-(\sigma^-)$ is black hole mass independent, T^q_{--} should also vanish at the past horizon ($\sigma^+ \to -\infty$) after taking $M \to 0$ limit. Then we find

$$t^\pm(\sigma^\pm) = -\frac{N\lambda^2\alpha^2}{16} \tag{25}$$

and one obtains

$$T^q_{--} \to 0 \tag{26}$$

at the future null infinity ($\sigma^+ \to +\infty$). Eqs.(24) and (26) might tell that there is no the Hawking radiation in the dilatonic supergravity model under discussion when quantum back-reaction of matter supermultiplet in large-N approach is taken into account. This might be the result of the positive energy theorem [11]. If the Hawking radiation is the positive and mass independent, the energy of the system becomes unbounded below. It could be also that since we work in strong coupling regime new methods to study Hawking radiation should be developed.

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